SALT STOCK MANAGEMENT BASED ON AN (R,S)-INVENTORY POLICY

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1. Abstract

In the past, winter maintenance in Flanders started in October with large fixed stocks. In total, approximately 38.000 tons of salt were stored in several depots at the beginning of the winter season. The stocks were built up during summertime at lower prices (up to 5% cheaper) and, correspond to the demand for salt during an 'average' winter period. In function of the salt usage and the contractual delivery times new salt was ordered during wintertime at winter prices. Only near the end of winter, an attempt was made to reduce the stocks. This policy certainly has some advantages: it is easy to implement and allows to operate relatively independent of the salt suppliers. However, its main drawback is that you may end up with large amounts of unused salt at the end of wintertime. These stocks have to be kept until the next winter period and will incur large holding costs.

To cope with this problem a new stock model based on an (R,S)-inventory policy is developed. The idea is to match the salt inventories in the depots closer to the actual demand for salt during the various winter months and so, in combination with shorter delivery times and better exchange of stock information between the salt suppliers and the road administration, reduce the stocks.
In an (R,S)-inventory policy, as soon as the stock reaches a minimal level R (reorder point), an order is placed to bring the stock to a maximum level S (stock level or target level). Both minimum and maximum stock level are not fixed during the whole winter period but will vary each month and for each district. The model parameters R and S are determined for a predefined service level and, in function of the historical spread and weather data. Two approximations are used. The first is based on a multi-linear regression (the weather type predictions made in the past are matched with the salt usage in the past corresponding to these weather types), the second approximation is based on pure historical data of salt amounts spread. The models allow decision support to know when and how much salt must be ordered. The first model is more accurate, the second one more easy to use. Both models still need further fine-tuning (weather data, based on daily data, …) but have proved to be useful. Also relevant financial gains seem to be possible.

Each district in the road administration will now, after the implementation of the model for the next winter and after feeding the model with the district specific data (amount of square meters to be spread, climatologic zone, safety parameters, …) manage his own salt stocks based on a uniform decision making process. The model must be fed with new (recent) data to become more and more accurate.

2. Introduction

Due to the greater financial responsibilities of each of the administrations within the Ministry of Flanders, the need to also rationalise the salt stock management led to an interesting investigation in co-operation with the Centre for Industrial Management of the University of Leuven.

3. Winter maintenance in Flemish Region

Competences regarding construction, management and maintenance of the main road network (motorways and regional ways) are assigned to the 3 Regions into which Belgium, as a federal state, has been divided: the Flemish, the Walloon and the Brussels-Capital Region. Due to this federalisation also winter maintenance on the main road network, as a service provided to the road user, is executed by these regions. In the Flemish Region, the management and the maintenance of their road network - 6.200 km roads, of which 1.300 km are motorways (slip roads included) - is done by 31 districts within 5 provinces. The Administration has been charged with the organisation, the decision making, the road inspection and the controlling of the execution which is done by private contractors. Each of the districts is rather self providing, but working in deliberation with the other districts. The districts manage their own salt stocks quite independently. In the past, this led to a slightly different approach of the inventory management and reordering policy in each of the districts.

In the past, winter maintenance in Flanders started in the second half of October with large fixed stocks. On the whole, approximately 38.000 tons of salt were stored in several depots at the beginning of the winter season. This amount generally corresponds to the demand for salt during an average winter period, although exceptionally, in a period of severe winter, it can be exhausted within six weeks. The stocks were compiled during summertime at lower prices (up to 5% cheaper) and represented a purchase price of 2,2 million Euro.

	Oct	Nov	Dec	Jan	Feb	Mar	Apr	**YEAR**
1985-86	11,13	209,04	112,74	397,37	137,65	90,19	0,06	**958**
1986-87	0	5,81	140,59	308,17	239,33	134,11	0	**828**
1987-88	0	3,29	47,78	21,84	85,80	93,21	0,03	**252**
1988-89	0,16	32,06	7,07	65,87	30,64	5,79	0,12	**142**
1989-90	0	17,13	58,92	12,75	7,08	26,87	0,18	**123**
1990-91	0	37,47	164,43	117,78	553,06	0,53	0,33	**874**
1991-92	0	23,86	39,07	64,16	44,41	0,29	0	**172**
1992-93	4,19	7,55	53,29	72,01	134,14	18,76	0,08	**290**
1993-94	0,56	194,14	77,92	73,61	153,55	0,03	2,67	**502**
1994-95	0	0	109,98	250,10	26,80	75,19	0,36	**462**
1995-96	0,10	32,51	248,26	133,37	246,84	25,74	2,31	**689**
1996-97	0	127,98	241,50	169,15	17,25	2,02	0,01	**558**
1997-98	0,02	6,85	43,03	59,10	49,59	7,82	0,01	**166**
1998-99	0	95,14	223,44	133,63	301,37	3,23	0,32	**757**
μ_{MONTH}	**1,15**	**56,63**	**112,00**	**134,21**	**144,82**	**34,56**	**0,46**	**484**
σ_{MONTH}	**3,08**	**71,48**	**80,36**	**112,52**	**150,79**	**44,37**	**0,87**	**298**

Table 1: Salt usage (in g/m²) for the Flemish Region

In function of the salt usage and the contractual delivery times new salt was ordered during wintertime at winter prices. Only near the end of winter, an attempt was made to reduce the stocks. This policy certainly has some advantages: it is easy to implement and allows operating relatively independent of the salt suppliers. However, its main drawback is that you may end up with large amounts of unused salt at the end of wintertime. These stocks have to be kept until the next winter period and will entail large holding costs. The greater financial responsibility made it necessary to investigate the possibility of saving money, without compromising the operational continuance and the safety on the roads.

4. Aim of the study

To cope with this problem a new stock model based on an (R,S)-inventory policy has been developed. The idea is to match the salt inventories in the depots and the actual demand for salt during the various winter months more closely and, consequently, to reduce the stocks, combining shorter delivery times with a better exchange of stock information between the salt suppliers and the road administration. A first indication that approvements in the general inventory policy were possible, was the comparative study of the relation between the treated surface and the inventory capacity of each district. Big differences showed up although no district (excepted Eeklo) had had big inventory problems in the last fifteen years.

	District	Surface (mio m²)	Hor. silos		Vert. silos		Total cap. (m³)	# turnouts
			#	Cap. (m³)	#	Cap. (m³)		
Antwerp	112-Ruisbroek	2,424	2	880	4	250	1.130	**28,0**
	114-Geel	2,842	2	800	0	0	800	**16,9**
	121-Antwerpen	4,814	2	770	3	285	1.055	**13,1**
	123-Brecht	1,902	1	600	0	0	600	**18,9**
	124-Grobbendonk	1,821	1	800	2	200	1.000	**32,9**
	125-Vosselaar	3,673	2	800	0	0	800	**13,1**
	Total for the province	**17,476**	**10**	**4.650**	**9**	**735**	**5.385**	**18,5**
Flemish-Brabant	211-Halle	1,616	2	880	3	375	1.255	**37,3**
	213-Herent	2,119	4	1.500	0	0	1.500	**34,0**
	221-Vilvoorde	4,607	1	1.000	3	450	1.450	**15,1**
	222-Bertem	2,722	1	900	2	515	1.415	**25,0**
	224-Aarschot	2,092	4	1.100	0	0	1.100	**25,2**
	Total for the province	**13,156**	**12**	**5.380**	**8**	**1.340**	**6.720**	**24,5**
West-Flanders	311-Brugge	2,293	3	950	0	0	950	**24,9**
	312 + 322 -Kortrijk	4,534	8	1.650	0	0	1.650	**21,8**
	313-Ieper	2,163	2	800	0	0	800	**22,2**
	314-Diksmuide	1,966	4	850	0	0	850	**25,9**
	315-Oostende	2,011	5	900	0	0	900	**26,9**
	316-Pittem	1,208	4	600	0	0	600	**29,8**
	321-Jabbeke	2,154	2	615	0	0	615	**17,1**
	Total for the province	**16,329**	**28**	**6.365**	**0**	**0**	**6.365**	**23,4**
East-Flanders	411-Gent	1,951	4	600	0	0	600	**18,5**
	412-Oudenaarde	2,350	4	600	0	0	600	**15,3**
	413-Eeklo	2,404	1	100	2	120	220	**5,5**
	414-Sint-Niklaas	2,819	3	1.200	1	65	1.265	**26,9**
	415-Aalst	1,834	1	800	2	125	925	**30,3**
	421-Gentbrugge	5,793	4	1.200	9	690	1.890	**19,6**
	Total for the province	**17,151**	**17**	**4.500**	**14**	**1.000**	**5.500**	**19,2**
Limburg	711-Sint-Truiden	2,377	1	600	3	175	775	**15,6**
	712-Hasselt	2,840	3	1.050	3	185	1.235	**20,9**
	713-Neerpelt	1,920	1	1.000	2	150	1.150	**28,8**
	714-Tongeren	1,757	2	800	5	240	1.040	**28,4**
	715-Maaseik	2,218	4	1.500	3	250	1.750	**37,9**
	716-Genk	3,420	2	1.000	4	315	1.315	**18,5**
	Total for the province	**14,532**	**13**	**5.950**	**20**	**1.315**	**7.265**	**24,0**
Total for the Flemish Region		**78,644**	**80**	**26.845**	**51**	**4.390**	**31.235**	**21,2**

Table 2 : Inventory capacity (Antwerp, W- and E-Flanders with 20 g/m², Fl-Brab and Limburg with 25 g/m²)

The main goal of a stock model is to be able to answer the following two questions: 'when shall we order?' and 'how much shall we order?'. In this case the choice for a certain kind of a model and the draft of the model itself, is not so obvious, due to the high degree of incertainty and variability of the demand for salt.

5. The (R,S)-inventory model

In an (R,S)-inventory policy, as soon as the stock reaches a minimal level R (reorder point), an order is placed to bring the stock to a maximum level S (target level or stock level) (cfr. figure 1). Both minimum and maximum stock levels are not fixed during the whole winter period but will vary each month and for each district. In this system an order is put as soon as the inventory position lowers to the reorder point. A variable quantity is ordered, just to allow the inventory position to reach the maximum level S, which has been calculated in terms of a predefined service level. After a time period LT (= delivery time or lead time) the order is available to fulfil the demand.

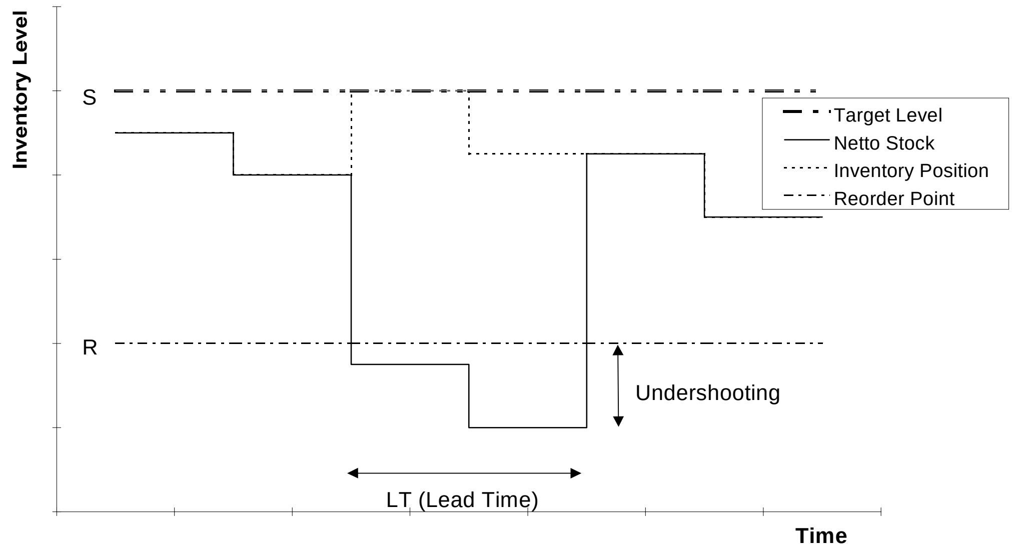


Figure 1: (R,S)-model

In a system of continuing follow-up an order can be put immediately after a demand of salt. Therefore it is necessary to put an order when the inventory level is still high enough to be safe during the delivery time. When the order is put exactly when the inventory position reaches the reorder level R, no shortage will occur during the delivery time if the total demand during this delivery time is lower than the reorder point. If the expected demand during the delivery time equals exactly R, and if there is a symmetric distribution of the demand during the delivery time, we can expect a shortage in half of the reordering cycles. So a buffer stock is added to ensure that the stock at the reorder point is higher than the expected demand during the delivery time and that the order is available before a shortage will occur. This buffer stock SS is calculated in function of a predefined service level and equals $k\sigma_{LT}$, in which k is a safety factor and σ_{LT} the standard deviation of the prediction errors on μ_{LT} (= the average expected demand during delivery time). The calculated values for SS, μ_{LT} and the expected monthly demand E(month) make it possible to deduce monthly values for the reorder level R and the maximum stock level S.

This model is to be preferred to some more common ones because it is based on a continuing follow-up and is useful in case of a variable demand. Taking into account that shortages are unacceptable for winter maintenance because of its important social and economic impact, preference is given to working with a service level model in which the predefined service level is put very high (namely 99,8%). Therefore no shortage costs were taken into account. The model parameters are determined by that predefined service level and in function of the delivery times and the historical spreading and weather data. Because of its simpleness, this specific model may be useful for all districts, after applying some correction factors if the district finds itself in an situation too different from the overall characteristics (for example: milder climate, other subsoil, strategic location). Two approximations are used.

6. First approximation

The first is based on a multi-linear regression. The weather type predictions made in the past are related to the salt usage in the past corresponding to these weather types. The weather predictions were delivered to the Flemish Region by the Meteorological Wing of the Belgian Air Forces, and the weather was identified by a letter ranging from A to G (= the wintertype). The further the letter ranks in the alphabet, the tougher the winter weather will be. Table 3 gives the average amount of days during a month for each of the wintertypes during the last 14 winters.

	Wintertype					
	A	**B**	**C**	**D**	**E**	**F**
Nov	**18,71**	**5,14**	**5,14**	**1,00**	**0,00**	**0,00**
Dec	**17,07**	**3,36**	**9,00**	**1,57**	**0,00**	**0,00**
Jan	**15,71**	**3,29**	**9,57**	**2,43**	**0,00**	**0,00**
Feb	**13,29**	**2,14**	**9,79**	**2,43**	**0,57**	**0,00**
Maart	**20,00**	**5,21**	**5,50**	**0,29**	**0,00**	**0,00**

Table 3: Average amount of days during a month for wintertypes A-F between 1985-1986 and 1999-2000

Because of the simpleness and also because the borders of the different climatic zones didn't correspond with these of the different districts, Flanders in a first approximation is counted as just one region for which only one model has been calculated (which can be made more specific by the use of teh correction factors, mentioned above). As for the spread data, relative values are used (these data are related to the surface to be treated, which can slightly differ year by year). With those relative values a regression equation was formulated for each of the winter months (November - March). In these equations the average salt amount spread during a week (= LT) or during a certain month is function of the amount of days a certain wintertype occurs in that period. For most of the equations the correlation lies above 0,7 which means that the wintertypes really play a major role, but some other parameters also play a minor role. In a first approximation they where neglected. The maximal deviation between the monthly spread data calculated by means of the regression equations and the real monthly spread data runs up to approximately 170 g/m². The average deviation is hardly 30 to 40 g/m², which corresponds approximately to some 1,5 to 2 spreading turnouts.

While calculating the different R- and S-values the following 4 assumptions were used: i.e. the undershooting is neglected, a month counts 4 independent weeks, the delivery time is 1 week and the onthly salt amounts spread are normally distributed.

		nov	dec	jan	feb	mar
(1)	**SS**	**57,69**	**64,14**	**84,28**	**117,12**	**49,94**
(2)	μ_{LT}	**15,84**	**28,15**	**28,04**	**35,36**	**8,25**
(3)=(1)+(2)	**R**	**73,53**	**92,29**	**112,32**	**152,48**	**58,19**
(4)	**E(month)**	**58,00**	**118,71**	**129,21**	**136,65**	**38,58**
(5)=(3)+(4)	**S**	**131,53**	**211,00**	**241,53**	**289,13**	**96,77**

Table 4: Parameter values (in g/m²) for the (R,S)-model based on the multilineair regression equations

In the table above, also the safety stock SS (= 2,88 σ_{LT}) and the average expected salt amount spread during a month E(month) are mentioned, for which 2,88 is the safety factor. This factor corresponds with a service level of 0,998 in case of a standard normal distribution. E(month), μ_{LT} and

σ_{LT} are calculated by means of the regression equations. Sometimes the reorder point is higher than the demand for that month which means that the expected demand for a month may have been used up within one week. This is - although exceptionally - proved by practice.

7. Second approximation

The second approximation is based on pure historical data of salt amounts spread. For this purpose the monthly data of 14 winters were the first to be brought in. Because in this approximation the demand distribution during delivery time must be known, the available montly data were used to simulate approximate weekly data (monthly data divided by 4).

	Oct	Nov	Dec	Jan	Feb	Mar	Apr
1985-86	**2,78**	**52,26**	**28,19**	**99,34**	**34,41**	**22,55**	**0,01**
1986-87	**0**	**1,45**	**35,15**	**77,04**	**59,83**	**33,53**	**0**
1987-88	**0**	**0,82**	**11,95**	**5,46**	**21,45**	**23,30**	**0,01**
1988-89	**0,04**	**8,02**	**1,77**	**16,47**	**7,66**	**1,45**	**0,03**
1989-90	**0**	**4,28**	**14,73**	**3,19**	**1,77**	**6,72**	**0,05**
1990-91	**0**	**9,37**	**41,11**	**29,44**	**138,27**	**0,13**	**0,08**
1991-92	**0**	**5,96**	**9,77**	**16,04**	**11,10**	**0,07**	**0**
1992-93	**1,05**	**1,89**	**13,32**	**18,00**	**33,54**	**4,69**	**0,02**
1993-94	**0,14**	**48,54**	**19,48**	**18,40**	**38,39**	**0,01**	**0,67**
1994-95	**0**	**0**	**27,50**	**62,52**	**6,70**	**18,80**	**0,09**
1995-96	**0,03**	**8,13**	**62,07**	**33,34**	**61,71**	**6,44**	**0,58**
1996-97	**0**	**31,99**	**60,37**	**42,29**	**4,31**	**0,50**	**0**
1997-98	**0,01**	**1,71**	**10,76**	**14,78**	**12,40**	**1,95**	**0**
1998-99	**0**	**23,79**	**55,86**	**33,41**	**75,34**	**0,81**	**0,08**

Table 5: Weekly demand (in g/m²) for Flemish Region

In this second approximation, the same 4 assumptions were used as in the first approximation. Even in the second appriximation Flanders is counted as one region, although here a province by province calculation could work. And for the incomplete months October and April a calculation can now also be done. Because in this appriximation the same service level of 0,998 is chosen, that salt amount spread for each month must be calculated which will fulfil the demand during delivery time in 99,8% of the cases. The results for the percentile 0,998 are mentioned in the table underneath.

		Oct	Nov	Dec	Jan	Feb	Mar	Apr
(1)	**SS**	**2,45**	**38,00**	**34,02**	**65,21**	**100,42**	**24,62**	**0,54**
(2)	**μ_{LT}**	**0,29**	**14,16**	**28,00**	**33,55**	**36,21**	**8,64**	**0,12**
(3)=(1)+(2)	**R**	**2,74**	**52,16**	**62,02**	**98,76**	**136,63**	**33,26**	**0,66**
(4)	**E(month)**	**1,15**	**56,63**	**112,00**	**134,21**	**144,82**	**34,56**	**0,46**
(5)=(3)+(4)	**S**	**3,89**	**108,79**	**174,02**	**232,97**	**281,45**	**67,82**	**1,12**

Table 6: Parameter values (in g/m²) for the (R,S)-model based on the direct technique

In the above table also the safety stock SS (= R - μ_{LT}) is mentioned. R is calculated for the precentile 0,998 and μ_{LT} and E(month) (=μ_{month}) are gathered from table 1 in which $\mu_{LT} = \mu_{month}/4$.

8. Conclusions of the study

The models allow decision support to know when and how much salt must be ordered. The first model is more accurate but needs a lot of data and time, the second one is more easy to use. The results of the second approximation prove to be about 20 gr/m² (more or less 1 spreading turnout) lower than in the first approximation. This means that in case of the second approximation someone will wait about 1 spreading turnout longer before reordering than in case of the first approximation. This is a consequence of the assumption that the weekly pattern is identical to the monthly one. Because experiences show that this is not exact and that the montly demand may be put into use in a very short period, this assumption is underestimating the possible demand during delivery time. Daily data must be substituted in the model. So, both models still need further fine-tuning but have proved to be useful. In any case relevant financial gains seem to be possible. The total stock capacity seems to be reducible with 20 to 25% which means substantical savings of holding costs.

To be able to see what results would have been obtained if the model (2^{nd} approximation) had been used in the past, a simulation was made for a random district. From this simulation we learned that with the actual values of the model parameters, shortages would have occurred now and then in the past. This, once again, shows the need of feeding the models with daily values.

9. Actual implementation

For each district the total surface of the roads that have to be treated has been calculated. Surfaces with porous asphalt counted double because they're also treated twice. The total surface for a district multiplied by the spread rate gives the salt use for one turnout in a district. The table beneath gives the reorder point R en the maximum stock level S, expressed in function of the amount. This table is based on the results of the second approximation. Following adaptations were put onto this table:

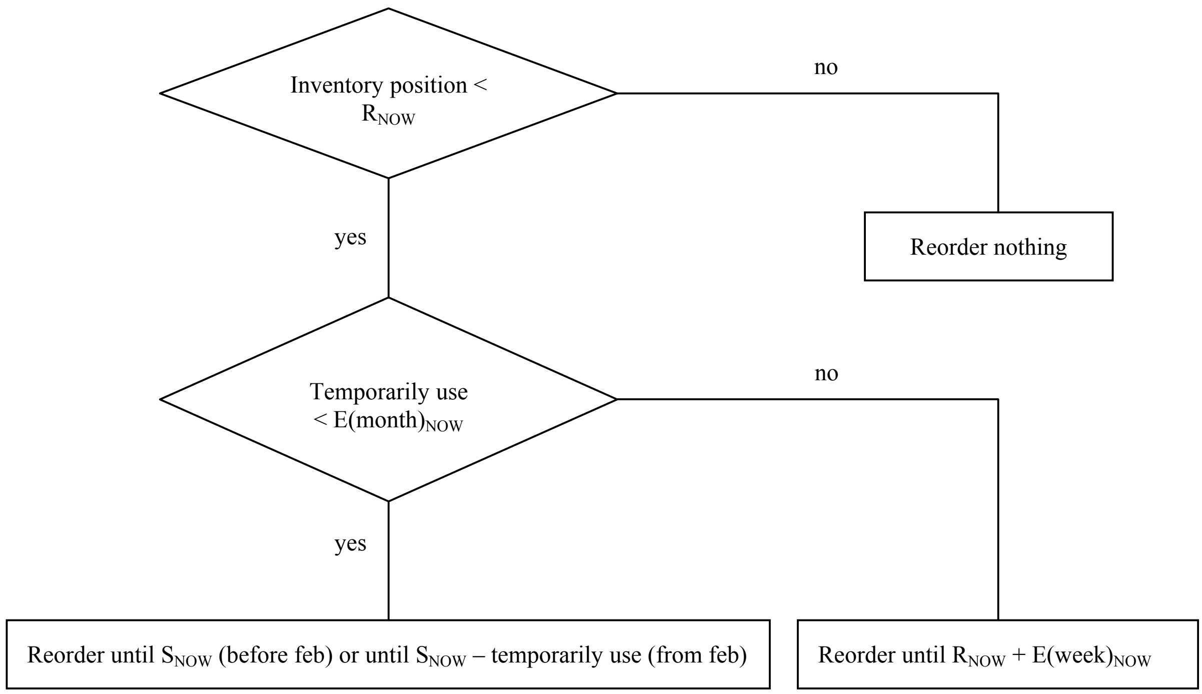
- 2,5 extra turnouts were adjusted because of underestimation (cfr. 6. and 8.)
- SS ≥ 4
- all figures were rounded up to an even (R) or an uneven figure (S) to facilitate interpolation
- the figures of January were raised to make them equal to those of February were taken.

	oct	nov	dec	jan	feb	mar	apr
R	**4**	**6**	**8**	**10**	**10**	**6**	**4**
S	**5**	**9**	**13**	**17**	**17**	**9**	**5**

Table 7: R- and S-values used for this winter in Flemish Region

The inventory position during a random month must be situated between these two values (R and S). This means that if the inventory position becomes lower than R, salt must to be reordered to bring the position again up to S. In the last week of each month (and in the last two weeks of December, because of the holiday period) the average R- en S-values of the actual and the next month must be taken into account.

WEEK 1 – 3:



WEEK 4:

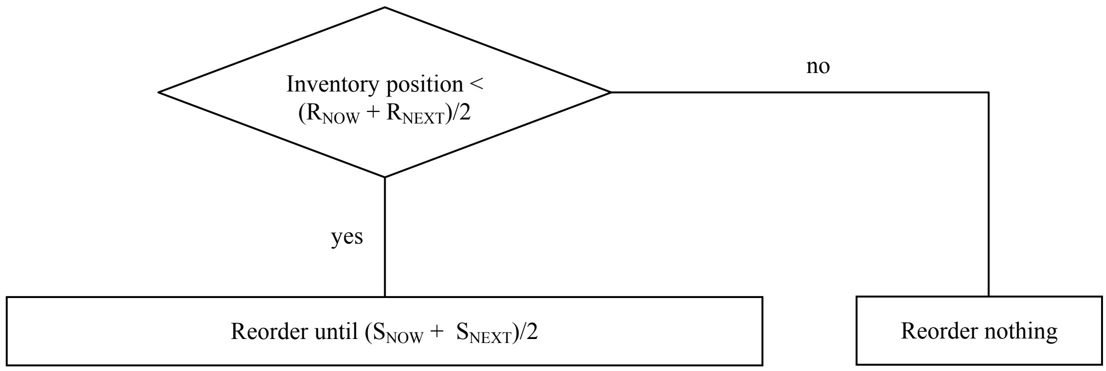


Figure 2: Decision making flow chart

The first of October the districts start with a restricted inventory position (the S-value for October or the end position of last winter if it was higher than S). This inventory position is kept at level and from the second half of December onwards, it will rise to its highest level, which will be kept until the end of February. From then on, the stock will be reduced in function of the weather circumstances, so that possibly at the end of april the S-level of 5 turnouts will be reached. Summer deliveries will no longer occur.

10. Future approvements

After the implementation of the model for the actual winter and after feeding the model with the specific data of the districts, each district in the Road Administration now will manage his own salt stocks based on a uniform decision making process. The model must be fed with new (recent) and more detailed (daily) data to become more and more accurate. Due to the results of the study the delivery times were shortened to 6 days and will once more be reduced to 4 or 5 days next year.

After this first investigation effectuated together with the University of Leuven, a second one has started. In this investigation a simulation model will be built up to be able to make a sensitivity analysis (influence of the variation of wintertype, delivery time, service level, …). Thus a refinement of the proposed inventory model and an evaluation of the financial and economical surplus value and of the service level of the refined model can be done, based on a statistical foundation.